**Static Dielectric**

We’ll take a look at a variety of thermal properties…

**Lindhardt Approximation to χirr(q)**

A more quantum mechanical approach to calculating the charge density response is the following. The introduction of the potential φ(r) will alter the free electron wavefunctions and since the ψ’s will change, we’ll have a change in the density. So we can calculate the charge density via (f is the statistical density matrix):



From perturbation theory, the first order correction to the wavefunction is (spin should be a spectator variable here):



Again we can construct the induced charge density,



Now,



where φ(p) refers to the Fourier transform of φ, and spin doesn’t matter. And also,



So then we can write:



where in the last we change variables q → q + k. Now in second term, change q → -q (and doing the spin sum),



In last line just changed q → k´. Now take the Fourier transform of both sides,



So then we have:



where in the last we change variables to k → k – q/2 in the first term, and k → k+q/2 in the second term. So finally, converting the sum to an integral:



First, let’s note that as q → 0, we have:



which matches the Thomas-Fermi result. Now let’s look at non-zero q. Should I bother? Well, at T = 0, and going back to a previous step, we have, changing variables k → -k in the second term (remember Jacobian of transformation is 1):



Now let’s evaluate this integral,



Align our kz axis with q, and then we can write:



Guess we could integrate by parts, and do a partial fraction decomposition or something (Well I did in the Single Particle Excitation file). But using online integrator, I get:



So then we get:



Note that the small x limit gives:



[Seems odd that if we consider the φ(r) = const. scenario, which corresponds to φ(q=0) and carefully redo the calculation, we’ll see that χirr(q=0) = 0. But if we use the formula above and take the limχirr(q→0) we don’t get zero. So it’s discontinuous. I think these calculations make sense only for weak local potentials] If we plot it, we have:

Chart, line chart

Description automatically generated

[Recall mkF/π2 = ρF in 3D] There is a logarithmic singularity in the derivative at x = 1 (q = 2kF). We’ll note that χirr(q), in the Linhardt approximation, approaches the Thomas-Fermi result in the small q limit (large r limit), as we’ll recall mkF/π2 is the 3D electron density of states at the Fermi surface. But it turns out the non-analyticity at q = 2kF, rather than the small q behavior, actually dominates the long r behavior.